

**B-Math-III Back paper Exam ; Differential Equations.**

Time : 2.00 hrs;      Max Mark: 45 ;      June 2022

1. Suppose that the steady state temperature distribution  $u(x, y)$  on a semi-infinite strip  $S := \{(x, y) : 0 \leq x \leq l, 0 \leq y < \infty\}$  has the boundary conditions  $u(0, y) = u(l, y) = 0, 0 < y < \infty$  and  $u(x, 0) = f(x), 0 < x < l$ . Find  $u(x, y)$  in  $S$ . (15)

2. Find the general solution of Legendre's equation

$$(1 - x^2)y'' - 2xy' + p(p + 1)y = 0,$$

where  $p$  is a constant. (15)

3. Find two linearly independent solutions of Euler's equation

$$x^2y'' + pxy' + qy = 0.$$

(15)

4. Suppose that the characteristic ODEs of an unknown first order, quasi-linear PDE with solution  $u(x, y)$  are given as

$$\frac{dx}{dt} = t + \frac{s}{2}; \frac{dy}{dt} = 1; \frac{du}{dt} = 1,$$

with the initial curve  $\Gamma_0 : \{(x, y); x = s, y = s, 0 \leq s \leq 1\}$ , and initial value  $u = \frac{s}{2}, 0 \leq s \leq 1$ . Find the PDE and solve it. (15)